Int J Theor Phys (2009) 48: 1090–1097
DOI 10.1007/s10773-008-9881-0

Tunneling of Charged and Magnetized Fermions from the Kerr-Newman-Ads Black Hole with Magnetic Charges

Xiao-Xiong Zeng · Qiang Li · Jia-Bo Hao

Received: 17 July 2008 / Accepted: 22 October 2008 / Published online: 30 October 2008

Abstract Tunneling of charged and magnetized Dirac particles from the Kerr-Newman-Ads black hole with magnetic charges is discussed in this paper. Owing to the electric and magnetic fields would couple with gravity field, we introduce the Dirac equation of charged and magnetized particles. Then by redefining the equivalent charge and gauge potential corresponding to the source with electric and magnetic charges, we discuss this tunneling once and obtain the same Hawking temperature. Both results show that the fermions tunneling formalism also come into existence in the charged and magnetized background space time.

Keywords Hawking radiation · Fermions · Tunneling · Kerr-Newman-Ads black hole · Magnetic charge

1 Introduction

In 1975, Hawking [1] came out with a striking discovery: if the quantum effect is taken into account, black holes are not black but can emit all kinds of particles in the form of pure thermal spectrum. Then in 1976, Damour and Ruffini [2] discussed emission of Klein-Gordon particles from the Schwarzschild and Kerr black hole. In the same year, Chandrasekhar [3] found the decoupled field equation of massive Dirac particles from the Kerr black hole and subsequently Page et al. [4] extended this work to the Kerr-Newman black hole successfully. The cases of extreme Kerr and Kerr-Newman black holes were also discussed in the following several years [5]. None of them, however, corresponded directly to one of the heuristic

X.-X. Zeng (✉)
Editorial Board of Journal of Sichuan University of Arts and Science, Dazhou 635000, People's Republic of China
e-mail: xxzengphysics@163.com

Q. Li · J.-B. Hao
Department of Physics and Engineering Technology, Sichuan University of Arts and Science, Dazhou 635000, People's Republic of China

Q. Li
e-mail: liqiang20040618@126.com

pictures that visualized the source of radiation as tunneling. Recently, Kerner and Mann [6] discussed the tunneling of Dirac particles from the Rindler space time and a general non-rotating black hole space time. Their work mainly based on the tunneling idea of Parikh and Wilczek [7] who discussed the tunneling of scalar particles from the spherical black hole. And their work also relied on the semi-classical WKB approximation, which relates the imaginary part of classical action that is related to the Boltzmann factor for emission at the Hawking temperature with the tunneling rate as

$$\Gamma \sim \exp\left(-\frac{2}{\hbar}\,\mathrm{Im}\,W\right) \tag{1}$$

implying one should first find action's imaginary part of emission for the sake of getting the tunneling probability. Unlike the case of scalar particles, where one employs the null geodesic equation that proposed by Parikh and Wilczek et al. [7–17] or the Hamilton-Jacobi equation that put forward by Agheben et al. [18–20] to ascertain the classical action, the action of fermions tunneling from inside to outside of horizon here is determined by the general Dirac equation. Until now, this work has been generalized to Reissner-Nordström black hole, where the Dirac equation of charged particles is introduced, by us [21] and to stationary space time in [22, 23] and dynamical black hole in [24].

In this paper, we attempt to further extend this method to discuss tunneling of charged and magnetized fermions from the Kerr-Newman-Ads black hole with magnetic charges. Several decades ago, Dirac predicted the existence of the magnetic monopole theoretically. But it was neglected due to the failure to detect such object in the following years. Recent years, the development of gauge theories [25, 26] has shed new light on the ingenious hypothesis and the string theory [27] also admits the existence of this subject. How to deal with the background with magnetic charge thus is very interesting and necessary. As far as the black hole with electric charge and magnetic charge is concerned, due to the magnetic field and electric field would couple with the gravity field and matter field, we introduce the Dirac equation of charged and magnetized particles to discuss tunneling of Dirac particles with electric and magnetic charges. Furthermore, we also provide a simplified skill to discuss the tunneling of charged and magnetized fermions. We find as the density ratio of electric charge to magnetic charge is constant and equals to that of the source, the electric and magnetic charges can be substituted by an equivalent charge. In this case, the tunneling of fermions with electric and magnetic charges can be simplified as that only with equivalent charge. This approach has been used in [28] to discuss the tunneling of charged and magnetized scalar particles. On the other hand, as for the AdS space time, it not only is interesting in the context of brane-world scenarios based on the setup of Randall and Sundrum but also plays major roles in the well-known AdS/CFT [29] conjecture. The correspondence between the supergravity in asymptotically AdS space times and CFT makes it possible to get some insights into the thermodynamic behavior of some strong coupling CFTs by studying thermodynamics of the asymptotically AdS space time [30]. Therefore investigation on tunneling of Dirac particles from black holes with electric and magnetic charges in AdS space time is of great significance and importance. Note that in this paper, to make the semi-classical WKB approximation available and choose the Gamma matrices properly, we discuss the particle tunneling in the dragging coordinate frame, where observers are co-rotating with the rotating black hole, and obtain the expected Hawking temperature.

The plan of this paper is the following. In Sect. 2 we describe the background geometry of Kerr-Newman-Ads black hole with magnetic charges and make the dragging coordinate transformation. Then in Sect. 3 we introduce Dirac equation of charged and magnetized particles and study its tunneling. Section 4 is devoted to our concluding remarks; particularly we redefine an equivalent charge and gauge potential to further simplify the above work.

2 Available Coordinate Transformation

The Kerr-Newman-Ads black hole with magnetic charges is an exact solution to the Einstein-Maxwell equations with a cosmological constant $\Lambda = -3/l^2$. Its explicit expression takes the form as [31, 32]

$$ds^2 = -\frac{\Delta_r}{\rho^2}\left(dt - \frac{a\sin^2\theta}{\Sigma}d\varphi\right)^2 + \frac{\rho^2}{\Delta_r}dr^2 + \frac{\rho^2}{\Delta_\theta}d\theta^2 + \Delta_\theta\frac{\sin^2\theta}{\rho^2}\left(adt - \frac{r^2+a^2}{\Sigma}d\varphi\right)^2, \quad (2)$$

where

$$\begin{aligned}\Delta_\theta &= 1 - \frac{a^2}{l^2}\cos^2\theta, \qquad \Delta_r = (r^2+a^2)\left(1+\frac{r^2}{l^2}\right) - 2m_0 r + q_e^2 + q_m^2,\\ \rho^2 &= r^2 + a^2\cos^2\theta, \qquad \Xi = 1 - \frac{a^2}{l^2},\end{aligned} \quad (3)$$

and the parameters m_0, a, q_e, q_m are related to the mass, angular momentum, electric and magnetic charges by the Komar integrals [32]

$$M = \frac{m_0}{\Xi^2}, \qquad J = \frac{am_0}{\Xi^2}, \qquad Q_e = \frac{q_e}{\Xi^2}, \qquad Q_m = \frac{q_m}{\Xi^2}. \quad (4)$$

The gauge potentials of electric field and magnetic fields are

$$\mathrm{A}_{\mu e} = -\frac{q_e r}{\rho^2}\left(dt - \frac{a\sin^2\theta}{\Xi}d\varphi\right), \qquad \mathrm{A}_{\mu m} = -\frac{q_m\cos\theta}{\rho^2}\left(adt - \frac{r^2+a^2}{\Xi}d\varphi\right). \quad (5)$$

There are four roots for $\Delta_r = 0$, but we are interested in the case when an event horizon appears, which means $m \geq m_{ext}$, where

$$\begin{aligned}m_{ext} = \frac{l}{3\sqrt{6}}&\left(\sqrt{\left(1+\frac{a^2}{l^2}\right)^2 + \frac{12}{l^2}(a^2+q_e^2+q_m^2)} + 2\frac{a^2}{l^2} + 2\right)\\ &\times\left(\sqrt{\left(1+\frac{a^2}{l^2}\right)^2 + \frac{12}{l^2}(a^2+q_e^2+q_m^2)} - \frac{a^2}{l^2} - 1\right)^{\frac{1}{2}}.\end{aligned} \quad (6)$$

Obviously, the event horizon and infinite red-shift surface of line element (2) are not coincident, the geometrical optics limit hence is not reliable at the horizon and the semi-classical WKB approximate is invalid. To solve this problem, we make the dragging coordinate transformation as $\phi = \varphi - \Omega t$, where

$$\Omega = -\frac{g_{03}}{g_{33}} = \frac{a\,\Xi[\Delta_\theta(r^2+a^2) - \Delta_r]}{\Delta_\theta(r^2+a^2)^2 - \Delta_r a^2\sin^2\theta}, \quad (7)$$

is the dragging velocity, line element (2) then changes as

$$ds^2 = -f dt^2 + h^{-1}dr^2 + pd\theta^2 + qd\varphi^2, \quad (8)$$

in which

$$f=\frac{\Delta_\theta\Delta_r\rho^2}{\Delta_\theta(r^2+a^2)^2-\Delta_r a^2\sin^2\theta},\qquad h=\frac{\Delta_r}{\rho^2},$$
$$p=\frac{\rho^2}{\Delta_\theta},\qquad q=\frac{\sin^2\theta[\Delta_\theta(r^2+a^2)^2-\Delta_r a^2\sin^2\theta]}{\rho^2\Xi^2}. \tag{9}$$

Under this transformation, the corresponding gauge potential of electric and magnetic fields only survive the temporal components

$$A_{\mu e}=A_{te}+A_{\phi e}=-\frac{\Delta_\theta q_e r(r^2+a^2)}{\Delta_\theta(r^2+a^2)^2-\Delta_r a^2\sin^2\theta}dt+\frac{q_e r a\sin^2\theta}{\rho^2\Xi}d\phi, \tag{10}$$

$$A_{\mu m}=A_{tm}+A_{\phi m}=-\frac{\Delta_r a q_m\cos\theta}{\Delta_\theta(r^2+a^2)^2-\Delta_r a^2\sin^2\theta}dt+\frac{q_m\cos\theta(r^2+a^2)}{\rho^2\Xi}d\phi. \tag{11}$$

For line element (8), the semi-classical WKB approximate can be used and it traits will also be embodied in choosing the Gamma matrices next.

3 Tunneling of Fermions with Electric and Magnetic Charge

Now, we focus on discussing tunneling of fermions with electric and magnetic charges from the Kerr-Newman-Ads background. As mentioned above, all tunneling approaches use the WKB approximation relating the tunneling probability with action's imaginary part of emission from the classically forbidden trajectory. As for the fermions in the electromagnetic field, we restore it to the following Dirac equation

$$\gamma^\mu\left(D_\mu-\frac{ieA_{\mu e}}{\hbar}-\frac{imA_{\mu m}}{\hbar}\right)\psi-\frac{im_0}{\hbar}\psi=0, \tag{12}$$

where the Greek indices $\mu,\nu=0,1,2,3$, e, m and m_0 being the electric charge, magnetic charge and mass of the fermions and

$$D_\mu=\partial_\mu+\Omega_\mu,\qquad \Omega_\mu=\frac{1}{2}i\Gamma^{\alpha\beta}_\mu\Sigma_{\alpha\beta},\qquad \Sigma_{\alpha\beta}=\frac{1}{4}i[\gamma^\alpha,\gamma^\beta]. \tag{13}$$

To solve the Dirac equation, we should first find the γ^μ matrices. Based on the relation $\{\gamma^\mu,\gamma^\nu\}=2g^{\mu\nu}I$, we pick for them as

$$\gamma^t=\frac{1}{\sqrt{f}}\begin{pmatrix} i & 0\\ 0 & -i\end{pmatrix},\qquad \gamma^r=\sqrt{h}\begin{pmatrix} 0 & \sigma^3\\ \sigma^3 & 0\end{pmatrix},$$
$$\gamma^\theta=\frac{1}{\sqrt{p}}\begin{pmatrix} 0 & \sigma^1\\ \sigma^1 & 0\end{pmatrix},\qquad \gamma^\varphi=\frac{1}{\sqrt{q}}\begin{pmatrix} 0 & \sigma^2\\ \sigma^2 & 0\end{pmatrix}, \tag{14}$$

in which σ^i $(i=1,2,3)$ is the Pauli Sigma matrices that follow

$$\sigma^1=\begin{pmatrix} 0 & 1\\ 1 & 0\end{pmatrix},\qquad \sigma^2=\begin{pmatrix} 0 & -i\\ i & 0\end{pmatrix},\qquad \sigma^3=\begin{pmatrix} 1 & 0\\ 0 & -1\end{pmatrix}. \tag{15}$$

Correspondingly, the matrix for γ^5 can be expressed as

$$\gamma^5 = i\gamma^t\gamma^r\gamma^\theta\gamma^\varphi = i\sqrt{\frac{h}{fpq}}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}. \tag{16}$$

Another necessity to solve the Dirac equation is the employment of wave function, here we make the following ansatz

$$\psi_\uparrow(t,r,\theta,\varphi) = \begin{bmatrix} A(t,r,\theta,\varphi)\xi_\uparrow \\ B(t,r,\theta,\varphi)\xi_\uparrow \end{bmatrix} \exp\left[\frac{i}{\hbar} I_\uparrow(t,r,\theta,\varphi)\right], \tag{17}$$

$$\psi_\downarrow(t,r,\theta,\varphi) = \begin{bmatrix} C(t,r,\theta,\varphi)\xi_\downarrow \\ D(t,r,\theta,\varphi)\xi_\downarrow \end{bmatrix} \exp\left[\frac{i}{\hbar} I_\downarrow(t,r,\theta,\varphi)\right], \tag{18}$$

where $I_{\uparrow/\downarrow}$ being action of the emitted spin up or down particles and $\xi_{\uparrow/\uparrow}$ being eigenvectors σ^3. In this paper, we are only interesting in the spin down case since the spin up is fully similar to this other than some changes of the sign. Inserting the wave function equation (18) into the Dirac equation (12), after Dividing by the exponential term and multiplying by $\hbar$, we find to leading order in $\hbar$

$$D\left(\frac{1}{\sqrt{p}}\partial_\theta I_\downarrow - \frac{i}{\sqrt{q}}(\partial_\varphi I_\downarrow + eA_{\varphi e} + mA_{\varphi m})\right) = 0, \tag{19}$$

$$-\frac{iC}{\sqrt{f(r,\theta)}}(\partial_t I_\downarrow - eA_{te} - mA_{tm}) + D\sqrt{h(r,\theta)}\partial_r I_\downarrow - m_0 C = 0, \tag{20}$$

$$C\left(\frac{1}{\sqrt{p}}\partial_\theta I_\downarrow - \frac{i}{\sqrt{q}}(\partial_\varphi I_\downarrow + eA_{\varphi e} + mA_{\varphi m})\right) = 0, \tag{21}$$

$$\frac{iD}{\sqrt{f(r,\theta)}}(\partial_t I_\downarrow - eA_{te} - mA_{tm}) + C\sqrt{h(r,\theta)}\partial_r I_\downarrow + m_0 D = 0. \tag{22}$$

Solving above equations directly is not easy due to the action is the function of coordinate components. However, taking into account the existence of time-like killing vector $(\frac{\partial}{\partial t})^a$ and space-like killing vector $(\frac{\partial}{\partial \varphi})^a$ in the stationary space time, we carry out the following separation variable

$$I_\downarrow = -(\omega - J\Omega)t + W(r,\theta), \tag{23}$$

where $\omega - J\Omega$ is the energy of radiation measured by observers in the dragging coordinate frame and $J$is the angular quantum number with respect to φ. Now, (19)–(22) can be rewritten as

$$D\left[\frac{1}{\sqrt{p}}W(r,\theta) - \frac{i}{\sqrt{q}}(J(\varphi) + eA_{\varphi e} + mA_{\varphi m})\right] = 0, \tag{24}$$

$$\frac{iC}{\sqrt{f(r,\theta)}}(\omega - J\Omega + eA_{te} + mA_{tm}) + D\sqrt{h(r)}W(r,\theta) + m_0 C = 0, \tag{25}$$

$$C\left[\frac{1}{\sqrt{p}}W(r,\theta) - \frac{i}{\sqrt{q}}(J(\varphi) + eA_{\varphi e} + mA_{\varphi m})\right] = 0, \tag{26}$$

$$-\frac{iD}{\sqrt{f(r,\theta)}}(\omega - J\Omega + eA_{te} + mA_{tm}) + C\sqrt{h(r,\theta)}W(r,\theta) + m_0 D = 0. \tag{27}$$

For (24) and (26), we find $J(\varphi)$ must be real due to $W(r,\theta)$ will be shown to be complex, implying $J(\varphi)$ contributes to the total tunneling probability little. On the other hand, for (25) and (27), we find no matter they decouple and couple

$$W_{\pm} = \pm \int \frac{\omega - J\Omega + eA_{te} + mA_{tm}}{\sqrt{f(r,\theta)h(r,\theta)}} dr, \tag{28}$$

must be true, where $+/-$ sign corresponds to the outgoing/ingoing mode. Integrating it directly, we find

$$W_{\pm} = \pm \pi i \frac{(\omega - \omega_0)(r_+^2 + a^2)}{r_+(1 + \frac{3r_+^2 + a^2}{l^2} - \frac{a^2 + q_e^2 + q_m^2}{r_+^2})}, \tag{29}$$

where $\omega_0 = \frac{aJ}{r_+^2 + a^2} + \frac{q_e r_+}{r_+^2 + a^2}$. The total tunneling rate hence can be expressed as

$$\Gamma \sim \frac{P(out)}{P(in)} = \frac{\exp(-2\,\mathrm{Im}\,W_+)}{\exp(+2\,\mathrm{Im}\,W_-)} = \exp(-4\,\mathrm{Im}\,W_+), \tag{30}$$

giving the expected Hawking temperature

$$T = \frac{r_+}{4\pi(r_+^2 + a^2)} \left(1 + \frac{3r_+^2 + a^2}{l^2} - \frac{a^2 + q_e^2 + q_m^2}{r_+^2}\right), \tag{31}$$

of the Kerr-Newman-Ads black hole with magnetic charge [33].

4 Concluding Remarks

In this paper, we discussed tunneling of fermions with electric and magnetic charges from the charged and magnetized stationary black hole in Ads background space time. We introduced the Dirac equation of charged and magnetized particles and obtain the expected Hawking temperature. In fact, we also can adopt the following skill [28] to deal with the magnetic charge so as to discuss conveniently.

Because the outside of black hole is an electromagnetic vacuum and electric and magnetic charges concentrate on the black hole, we regard it as a conducting sphere. The electromagnetic tensor is

$$F_{\mu\nu} = \nabla_\nu A_\mu - \nabla_\mu A_\nu + G^+_{\mu\nu}, \tag{32}$$

where $G^+_{\mu\nu}$ is the Dirac string term. The Maxwell equations read off

$$\nabla_\nu F^{\mu\nu} = 4\pi\rho_e u^\mu, \tag{33}$$

$$\nabla_\nu F^{+\mu\nu} = 4\pi\rho_m u^\mu, \tag{34}$$

where $F^{+\mu\nu}$ is the dual tensor of $F^{\mu\nu}$, ρ_e and ρ_m represent the densities of electric and magnetic charges, while u^μ stand for the 4-velocity. If we define

$$\tilde{F}^{\mu\nu} = F^{\mu\nu}\cos\beta + F^{+\mu\nu}\sin\beta, \tag{35}$$

where β denotes a real constant angle. Equations (33) and (34) will change as

$$\nabla_\nu \tilde{F}^{\mu\nu} = 4\pi(\rho_e \cos\beta + \rho_m \sin\beta)u^\mu, \tag{36}$$

$$\nabla_\nu \tilde{F}^{+\mu\nu} = 4\pi(-\rho_e \sin\beta + \rho_m \cos\beta)u^\mu. \tag{37}$$

Letting

$$\rho_e \cos\beta + \rho_m \sin\beta = \rho_h, \tag{38}$$

$$-\rho_e \sin\beta + \rho_m \cos\beta = 0, \tag{39}$$

besides yields $\rho_e/\rho_m = \cot\beta$, the Maxwell equations can be simplified as

$$\nabla_\nu \tilde{F}^{\mu\nu} = 4\pi\rho_h u^\mu, \tag{40}$$

$$\nabla_\nu \tilde{F}^{+\mu\nu} = 0, \tag{41}$$

where $\tilde{F}_{\mu\nu} = \nabla_\nu \tilde{A}_\mu - \nabla_\mu \tilde{A}_\nu$ and the corresponding general coordinate are

$$\tilde{A}_\mu = (\tilde{A}_0, \tilde{A}_1, \tilde{A}_2, \tilde{A}_3). \tag{42}$$

Now, the Maxwell equations are simplified as the source only with equivalent charge. Therefore, if we consider the black hole as a conducting sphere while the electric charge and the magnetic charge concentrate on the black hole with the density rate as $\rho_e/\rho_m = \cot\beta$, we have

$$q_h^2 = q_e^2 + q_m^2, \tag{43}$$

where q_h stand for the equivalent charge corresponding to the density ρ_h. In this case, the electromagnetic vector potential in the dragging coordinate frame is

$$\tilde{A}_\mu = \left(-\frac{\Delta_\theta q_h r(r^2+a^2)}{\Delta_\theta(r^2+a^2)^2 - \Delta_r a^2 \sin^2\theta}, 0, 0, 0\right) \tag{44}$$

and the Dirac equation also should be changed as

$$\gamma^\mu\left(D_\mu - \frac{iq_h\tilde{A}_\mu}{\hbar}\right)\psi - \frac{im_0}{\hbar}\psi = 0. \tag{45}$$

Inserting the wave function (18), we find the action of charged and magnetized fermions is

$$W_\pm = \pm\pi i \frac{(\omega-\omega_0)(r_+'^2+a^2)}{r_+'(1+\frac{3r_+'^2+a^2}{l^2} - \frac{a^2+q_h^2}{r_+'^2})}, \tag{46}$$

where r_+' satisfies $\Delta_r' = (r_+'^2+a^2)(1+\frac{r_+'^2}{l^2}) - 2m_0 r_+' + q_h^2 = 0$ and $\omega_0 = \frac{aJ}{r_+'^2+a^2} + \frac{q_h r_+'}{r_+'^2+a^2}$.

Considering (43), we also get the same emission temperature as in (31). Surely, as the self-gravitational interaction and back reaction of radiant spin particles are considered [18–20], we also can find correction to this temperature.

Acknowledgement This work is supported by the natural science foundation of Sichuan education office under grant number 07ZC039.

References

1. Hawking, S.W.: Math. Phys. **43**, 199 (1975). doi:10.1007/BF02345020
2. Damour, T., Ruffini, R.: Phys. Rev. D Part. Fields **14**, 332 (1976). doi:10.1103/PhysRevD.14.332
3. Chandrasekhar, S.: Proc. R. Soc. Lond. A **349**, 541 (1976)
4. Page, D.N.: Phys. Rev. D Part. Fields **14**, 1509 (1976). doi:10.1103/PhysRevD.14.1509
5. Liu, L., Xu, D.Y.: Acta Phys. Sin. **29**, 1617 (1980) (in Chinese)
6. Kerner, R., Mann, R.B.: Class. Quantum Gravity **25**, 095014 (2008). doi:10.1088/0264-9381/25/9/095014
7. Parikh, M.K., Wilczek, F.: Phys. Rev. Lett. **85**, 5042 (2000). doi:10.1103/PhysRevLett.85.5042
8. Zhang, J.Y., Zhao, Z.: Phys. Lett. B **618**, 14 (2005). doi:10.1016/j.physletb.2005.05.024
9. Zhang, J.Y., Zhao, Z.: Nucl. Phys. B **725**, 173 (2005). doi:10.1016/j.nuclphysb.2005.07.024
10. Yang, S.Z.: Int. J. Theor. Phys. **45**, 965 (2006). doi:10.1007/s10773-006-9092-5
11. Jiang, Q.Q., Wu, S.Q.: Phys. Lett. B **635**, 151 (2006). doi:10.1016/j.physletb.2006.01.035
12. Jiang, Q.Q., Wu, S.Q., Cai, X.: Phys. Rev. D Part. Fields Gravit. Cosmol. **76**, 064003 (2006). doi:10.1103/PhysRevD.73.064003
13. Vagenas, E.C.: Phys. Lett. B **503**, 399 (2001). doi:10.1016/S0370-2693(01)00242-8
14. Vagenas, E.C.: Phys. Lett. B **584**, 127 (2004). doi:10.1016/j.physletb.2004.01.039
15. Zeng, X.X., Hou, J.S., Yang, S.Z.: Pramana J. Phys. **70**, 409 (2008)
16. Liu, C.Z., Zhang, J.Y., Zhao, Z.: Phys. Lett. B **639**, 670 (2006). doi:10.1016/j.physletb.2006.07.023
17. Liu, W.B.: Phys. Lett. B **634**, 541 (2006). doi:10.1016/j.physletb.2006.01.028
18. Agheben, M., Nadalini, M., Vanzo, L., Zerbini, S.: J. High Energy Phys. **0505**, 014 (2005)
19. Medved, A.J.M., Vagenas, E.: Mod. Phys. Lett. A **20**, 2449 (2005). doi:10.1142/S021773230501861X
20. Chen, D.Y., Yang, S.Z.: Gen. Relativ. Gravit. **39**, 1503 (2007). doi:10.1007/s10714-007-0478-3
21. Zeng, X.X., Yang, S.Z.: Gen. Relativ. Gravit. **40**, 2107 (2008). doi:10.1007/s10714-008-0618-4
22. Li, R., Ren, J.R.: Phys. Lett. B **661**, 370 (2008). doi:10.1016/j.physletb.2008.01.077
23. Chen, D.Y., Jiang, Q.Q., Zu, X.T.: Phys. Lett. B **665**, 106 (2008). doi:10.1016/j.physletb.2008.05.064
24. Criscienzo, R.D., Vanzo, L.: Europhys. Lett. **82**, 60001 (2008). doi:10.1209/0295-5075/82/60001
25. ’t Hooft, G.: Nucl. Phys. B **79**, 276 (1974). doi:10.1016/0550-3213(74)90486-6
26. Schwinger, J.S.: Phys. Rev. **144**, 1087 (1966). doi:10.1103/PhysRev.144.1087
27. Mignemi, S.: Phys. Rev. D Part. Fields **51**, 934 (1995). doi:10.1103/PhysRevD.51.934
28. Zhang, J.Y., Fan, J.H.: Phys. Lett. B **648**, 133 (2007). doi:10.1016/j.physletb.2007.03.006
29. Hawking, S.W., Hunter, C.J., Taylor, M.: Phys. Rev. D Part. Fields **59**, 064005 (1999). doi:10.1103/PhysRevD.59.064005
30. Caldarelli, M.M., Cognola, G., Klemm, D.: Class. Quantum Gravity **17**, 399 (2000). doi:10.1088/0264-9381/17/2/310
31. Carter, B.: Commun. Math. Phys. **10**, 280 (1968)
32. Belgiorno, F., Cacciatori, S.L.: arXiv:0803.2496 [math-ph]
33. Jiang, Q.Q., Wu, S.Q., Cai, X.: Phys. Lett. B **647**, 200 (2007). doi:10.1016/j.physletb.2007.01.058